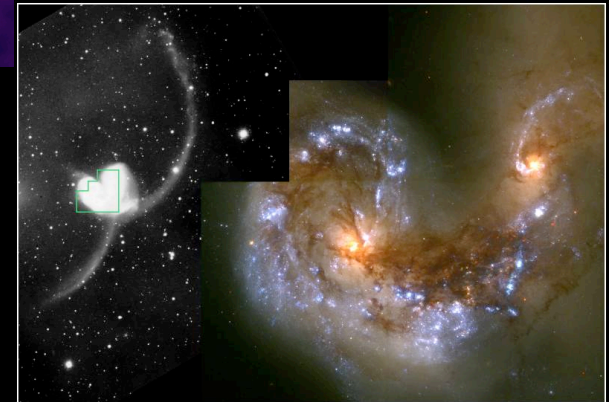
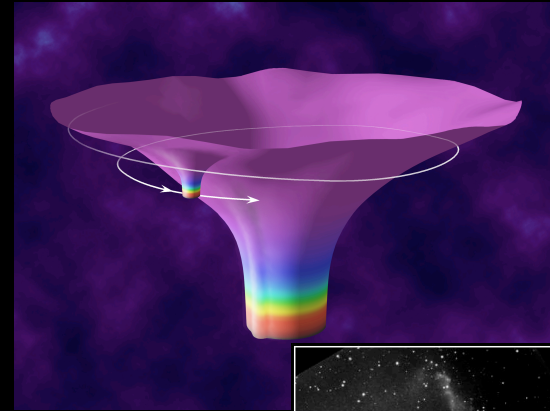


# 'astrophysics tutorial'

A quick walk through  
LISA sources

What gravitational  
waves are uniquely  
suited to teach us  
about these systems.



Colliding Galaxies NGC 4038 and NGC 4039 HST • WFPC2  
PRC97-34a • ST ScI OPO • October 21, 1997 • B. Whitmore (ST ScI) and NASA



# The electromagnetic universe: Dynamics of charges and plasmas

Leading order effect is  
time variation of  
charge dipole moment:

Strong coupling!  
Coefficient of  
radiation field is 1  
in “correct” units.

$$d_a = \int \rho_q(x') x'_a d^3x'$$

$$A_a \simeq \frac{\dot{d}_a}{r} \sim \frac{qv}{r}$$

Strong coupling means  
radiation is readily  
created, (relatively)  
easily measured.



# The gravitational wave universe: Things that are dense and relativistic

At leading order, GR teaches us that GWs are generated by the time variation of a source's "mass" quadrupole moment.

$$Q_{ab} = \int \rho_m(x') x'_a x'_b d^3x'$$
$$h_{ab} \simeq \frac{G}{c^4} \frac{\ddot{Q}_{ab}}{r} \simeq \frac{G}{c^4} \frac{mv^2}{r}$$

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Need large  $m$  and large  $v$  to compensate for smallness of  $G/c^4$  !

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Need large  $m$  and large  $v$  to compensate for smallness of  $G/c^4$  !

## Direct probe of spacetime dynamics

# Organization of sources

Convenient way to categorize sources:  
By search technique. Each broad category  
presents a different data analysis challenge,  
requires a very different approach.

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## Stochastic backgrounds

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Stochastic backgrounds

Periodic sources



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Chirping sources

# Organization of sources

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Stochastic backgrounds

Periodic sources

Chirping sources

*Really complicated chirping sources*

# Cosmological stochastic backgrounds

Concordance model indicates early universe underwent rapid *inflation* driven by a scalar field:

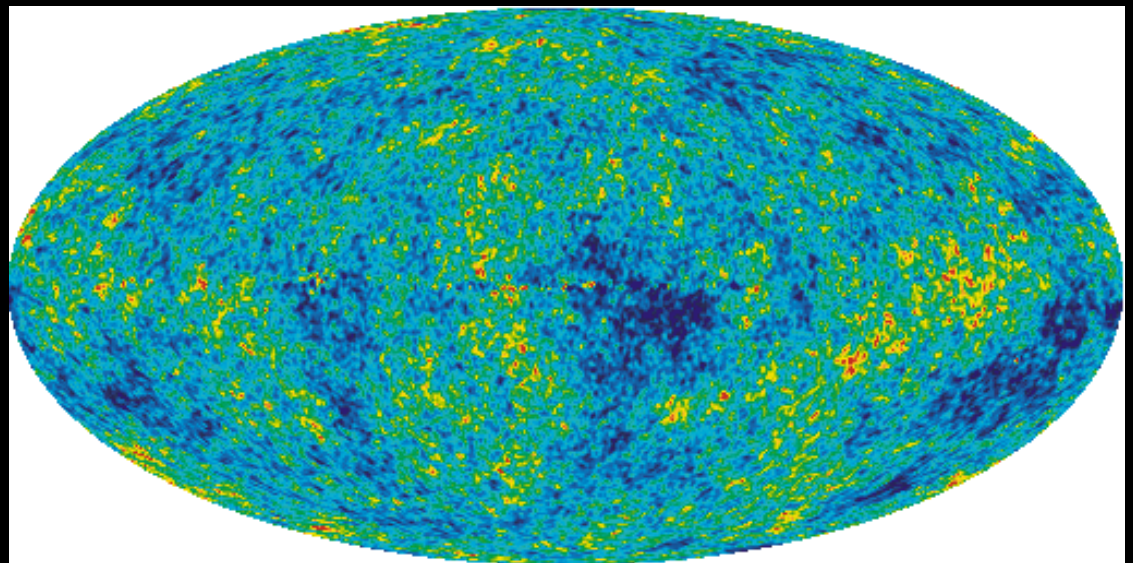
$$\ddot{a} = \frac{8\pi G V(\phi)}{3} a$$

→ Exponential growth

*Zero point fluctuations* in that field seeded density inhomogeneities

... gives inhomogeneities in the gravitational potential

... leads to fluctuations in the temperature of the cosmic microwave background.



Stunningly good agreement between model and observations!

# Cosmological stochastic backgrounds

Spacetime metric also experiences  
zero-point fluctuations!

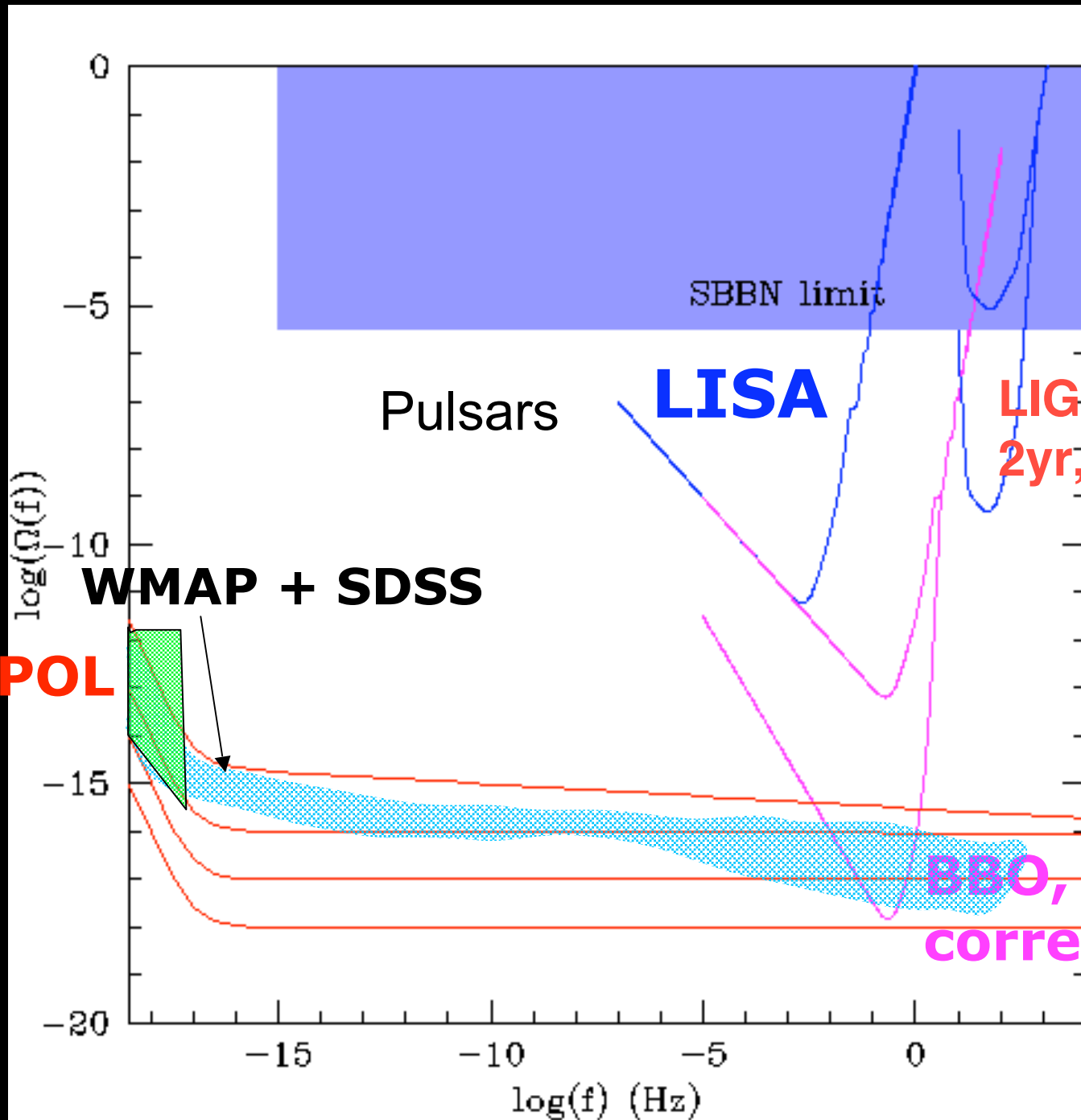
$$g_{ab} = g_{ab}^{\text{FRW}} + h_{ab}(t, \vec{r})$$

Friedmann-Robertson-  
Walker background

Fluctuations

Measurement of these waves would allow a  
direct reconstruction of the inflationary  
potential: *Direct probe of inflation physics.*

# Predictions of least contrived scale-free inflation models



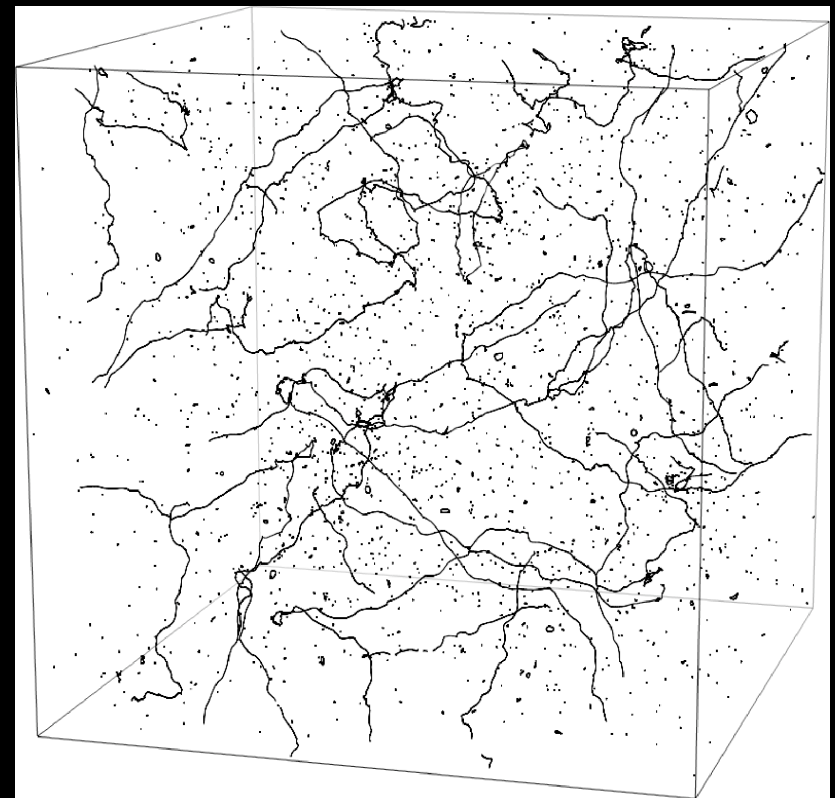
(Slide  
stolen  
from  
Phinney)

# Cosmological stochastic backgrounds

As universe cooled, underwent phase transitions as unified interactions separated.

Other models predict production of cosmic strings via phase transitions or during the condensation of our 3 dimensional brane from the early universe dynamics.

Graphic: Bruce Allen,  
University of Wisconsin-Milwaukee



Strings whirl around at relativistic speeds - can produce GWs much like the cracking of a whip.



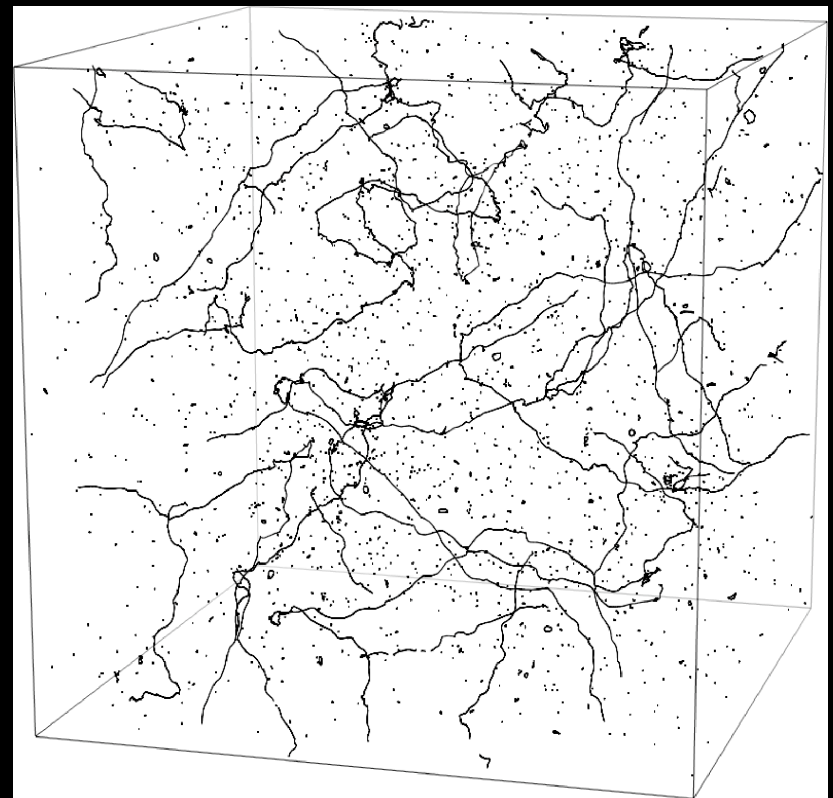
# Cosmological stochastic backgrounds

As universe cooled, underwent phase transitions as unified interactions separated.

Other models predict production of cosmic strings via phase transitions or during the condensation of our 3 dimensional brane from the early universe dynamics.

Current results from string theory predict a spectrum that *could* be peaked right in the LISA band.

Graphic: Bruce Allen,  
University of Wisconsin-Milwaukee



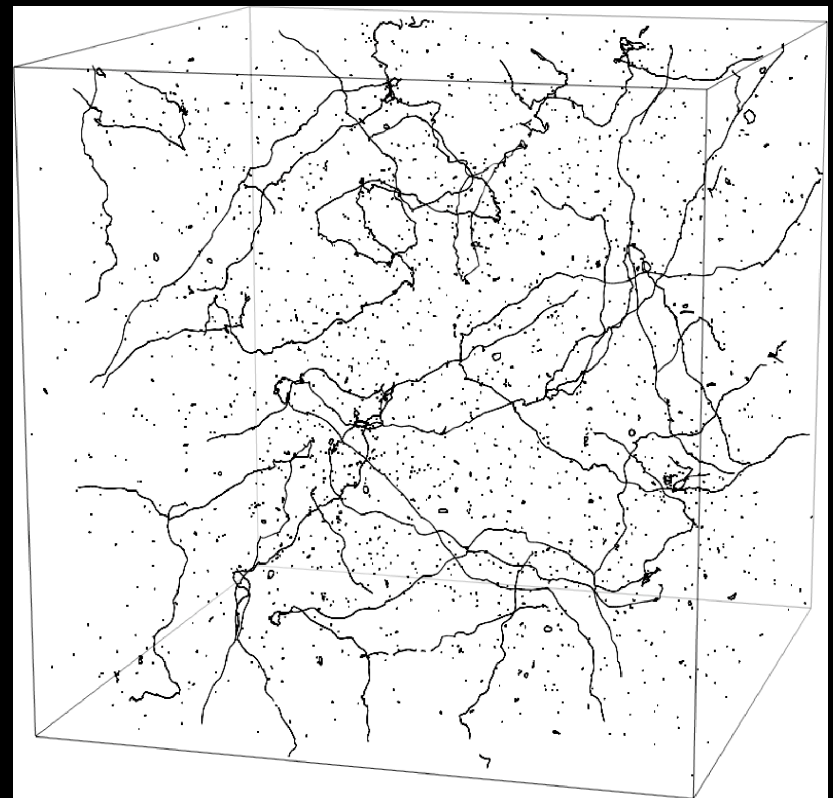
# Cosmological stochastic backgrounds

As universe cooled, underwent phase transitions as unified interactions separated.

Models *can* produce signals in LISA band, *if* we get lucky.

Example: Electroweak phase transition expected to have spectrum peaked in LISA band. *If* transition is strongly first order, amplitude will be measureable.

Graphic: Bruce Allen,  
University of Wisconsin-Milwaukee



## Punchline:

LISA has ***NO CHANCE*** of measuring inflationary waves: Simply too weak!

Cosmological background would require us to get lucky with respect to phase transitions; or, realizations of somewhat more speculative ideas.

A lot of discovery space!

# Periodic sources

In LISA band: binary star systems, mostly white dwarf binaries.

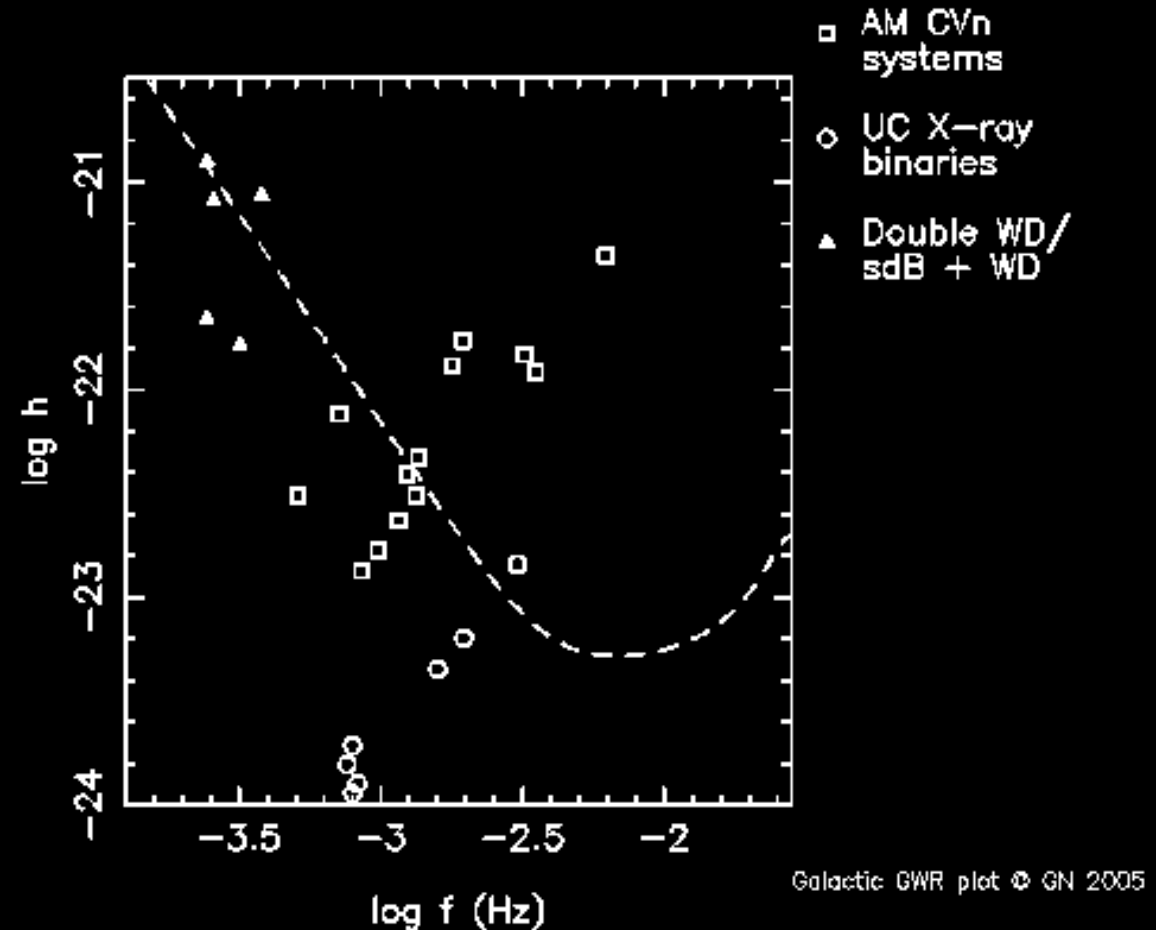
$$\dot{f} = \frac{48}{5\pi} \mathcal{M}^{5/3} (2\pi f)^{11/3}$$

In mass and frequency bands of interest ( $M \sim 0.5 - 1 M_{\text{sun}}$ ,  $f \sim 10^{-4} - 10^{-2}$  Hz),  $f$  changes very little over a multiyear LISA mission - sources are essentially monochromatic.

# Periodic sources

## Some sources guaranteed!

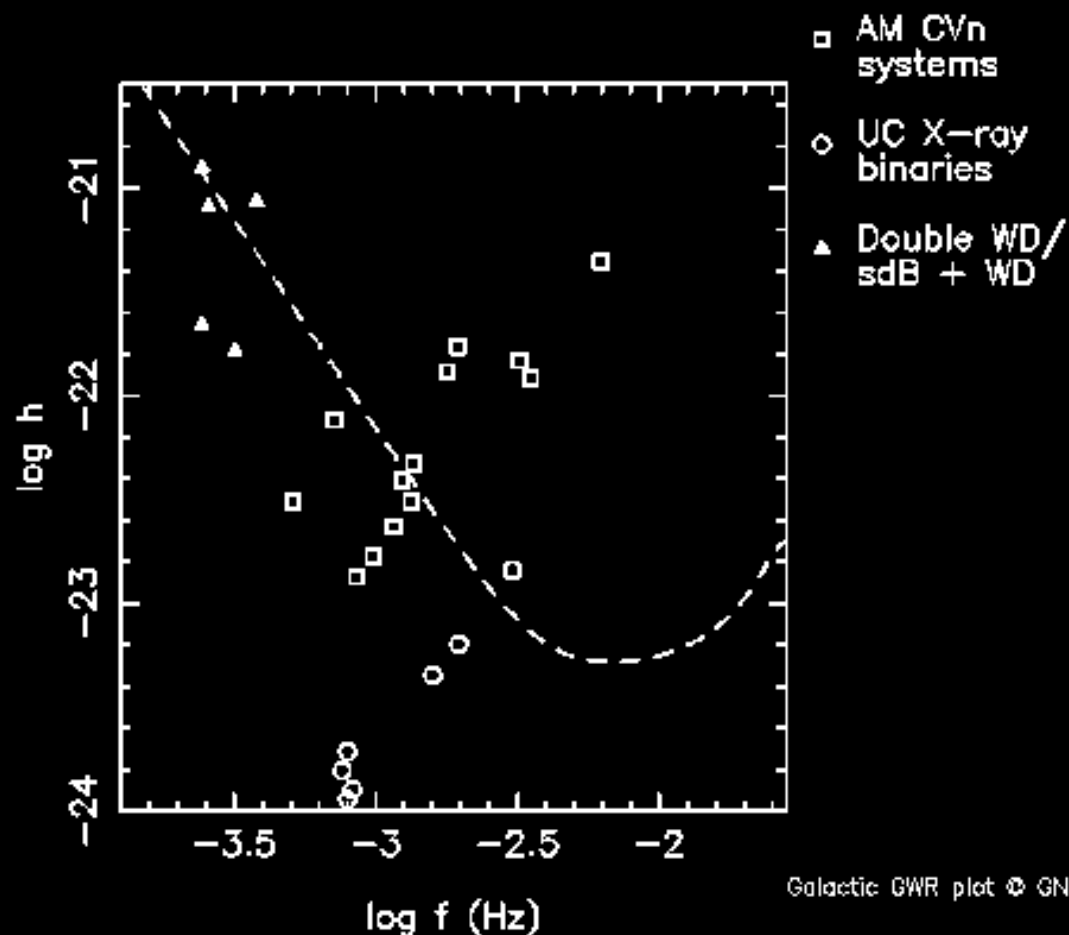
Plot courtesy  
Gijs Nelemans,  
shows known  
compact systems  
that radiate in  
the LISA band.



# Periodic sources

## Some sources guaranteed!

Current status  
not so well  
constrained as  
indicated here:  
E.g., distance &  
inclination not  
well known.

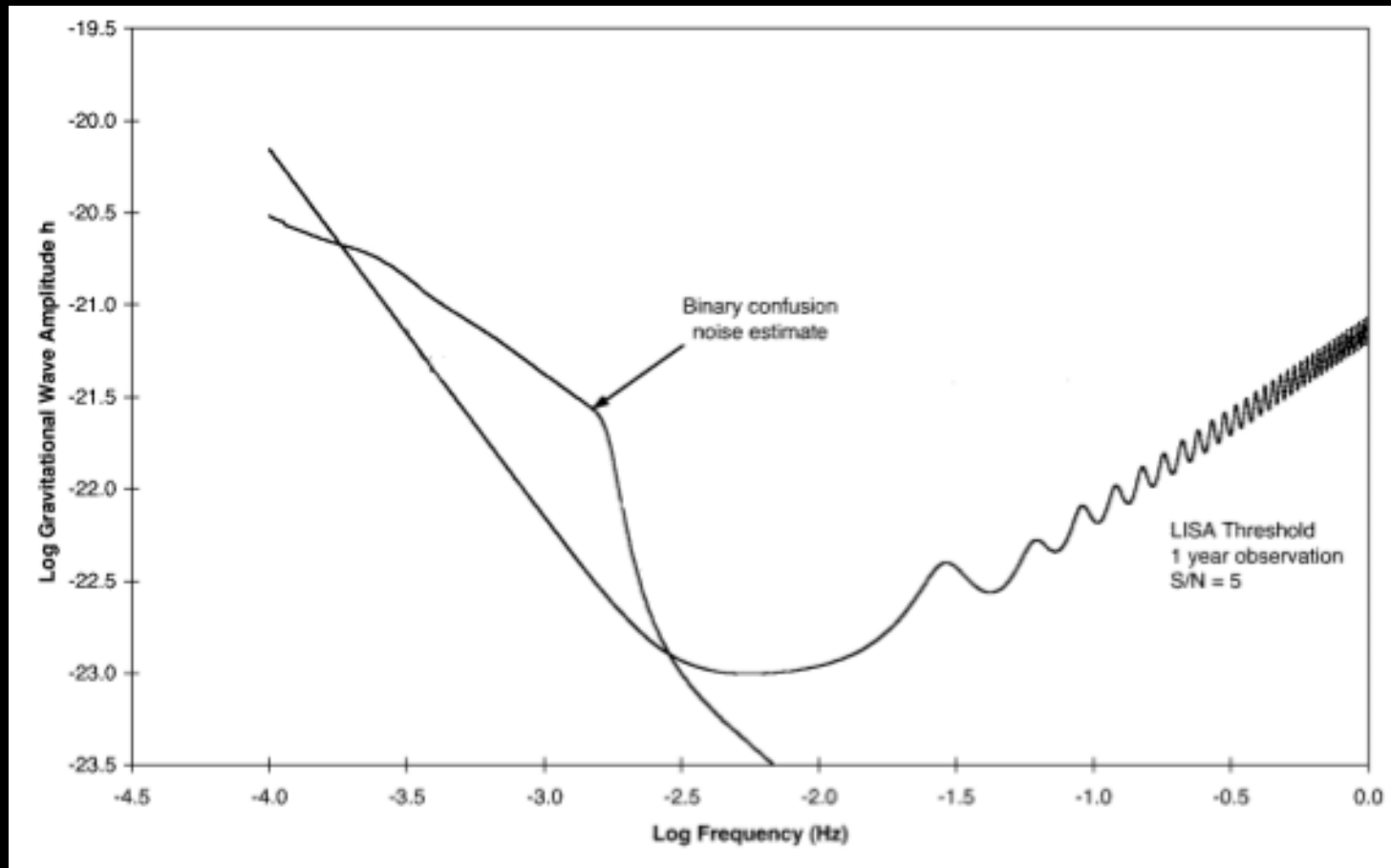


Combined GW/EM observations particularly  
powerful with these sources.



# Periodic 4 Stochastic

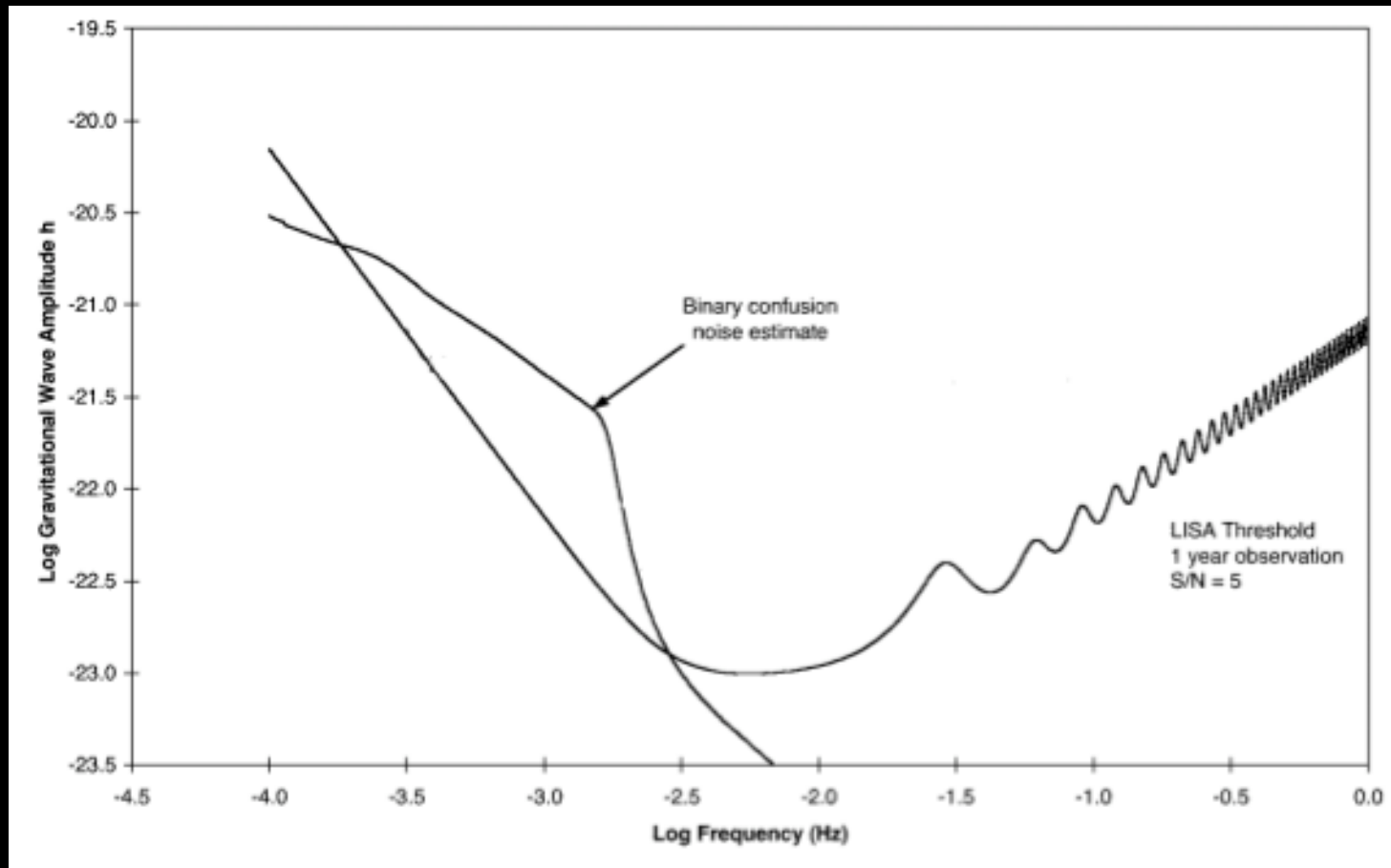
Add model for the distribution of galactic binaries ... so numerous that they overlap and form a background!



“Noise”  
from the  
viewpoint  
of studying  
other GW  
sources...

# Periodic 4 Stochastic

Add model for the distribution of galactic binaries ... so numerous that they overlap and form a background!



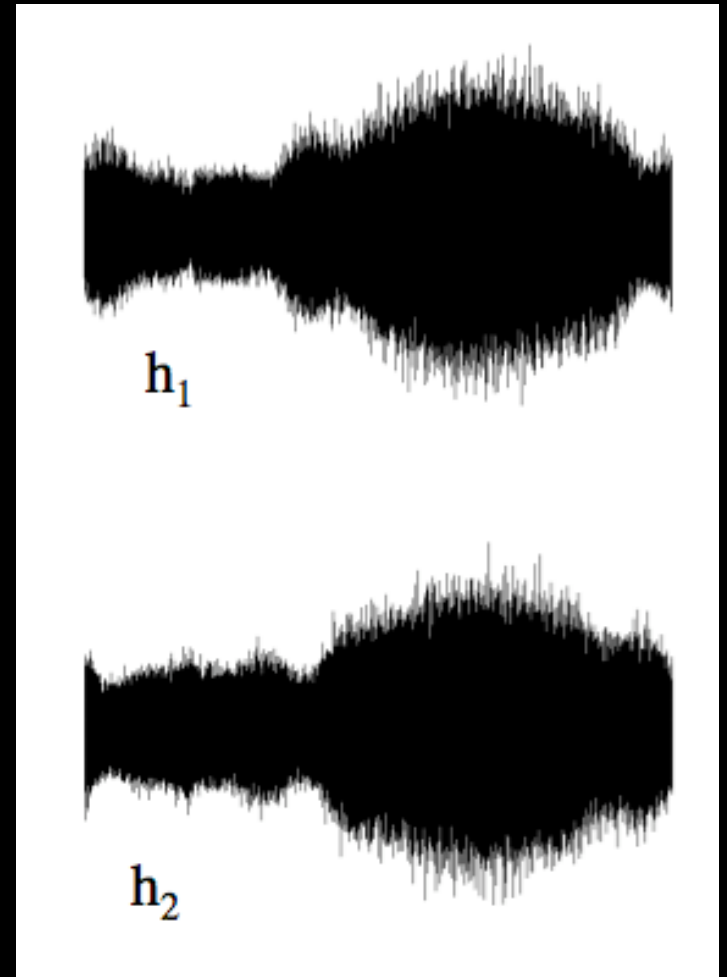
“Signal” if  
stellar  
populations  
is what you  
want to  
know  
about!

# Example: Contribution to background of globular cluster NGC 104 (47 Tuc)

Nearby globular cluster with a dense core; contains many binary systems and millisecond pulsars.

Position of cluster + motion of detector imposes unique modulation on waves.

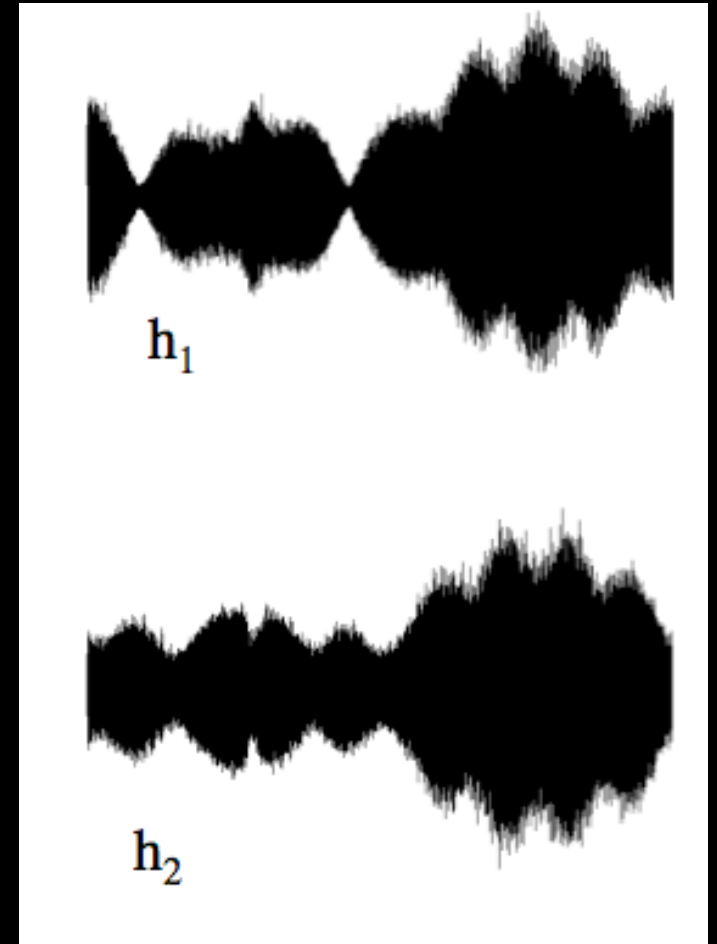
Timeseries shown here!



Population synthesis  
by Matt Benacquista

# Example: Contribution to background of globular cluster NGC 6752

Also nearby, core-collapsed, very high binary fraction in the core (~15 - 40%).



Timeseries  
shown here!

Population synthesis  
by Matt Benacquista

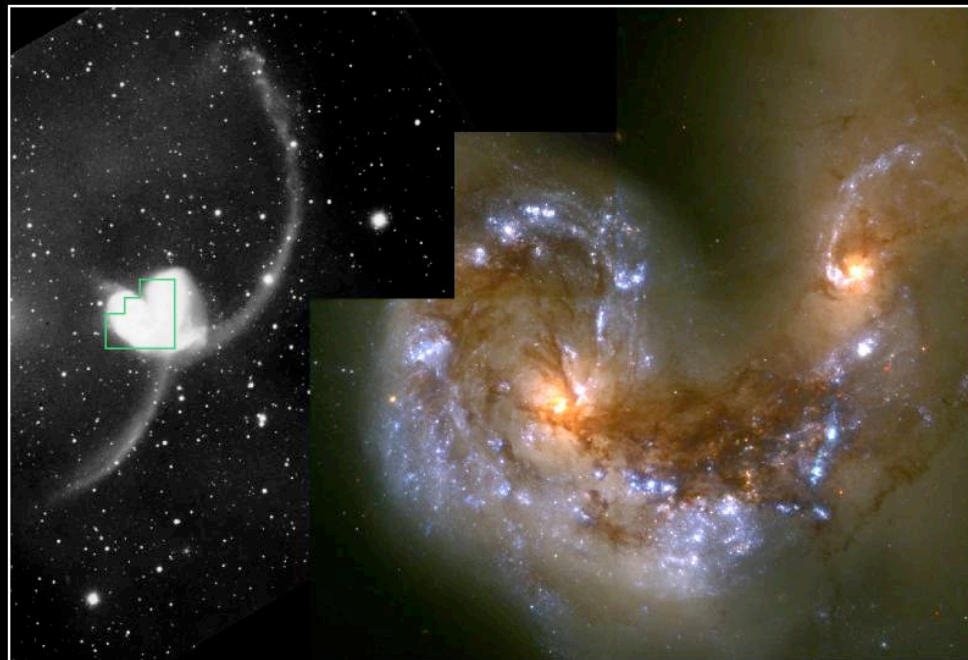
# Chirping sources: massive BH binaries

Massive binaries produced by *galaxy and halo mergers*.

Mergers are common, especially in the past.

Now known that almost *all* galaxies host a massive black hole ...

**Binary black hole formation is *frequent* at these masses!**



Colliding Galaxies NGC 4038 and NGC 4039 HST • WFPC2  
PRC97-34a • ST ScI OPO • October 21, 1997 • B, Whitmore (ST ScI) and NASA

Interesting event rate almost certain: At least a few events per year; perhaps hundreds.

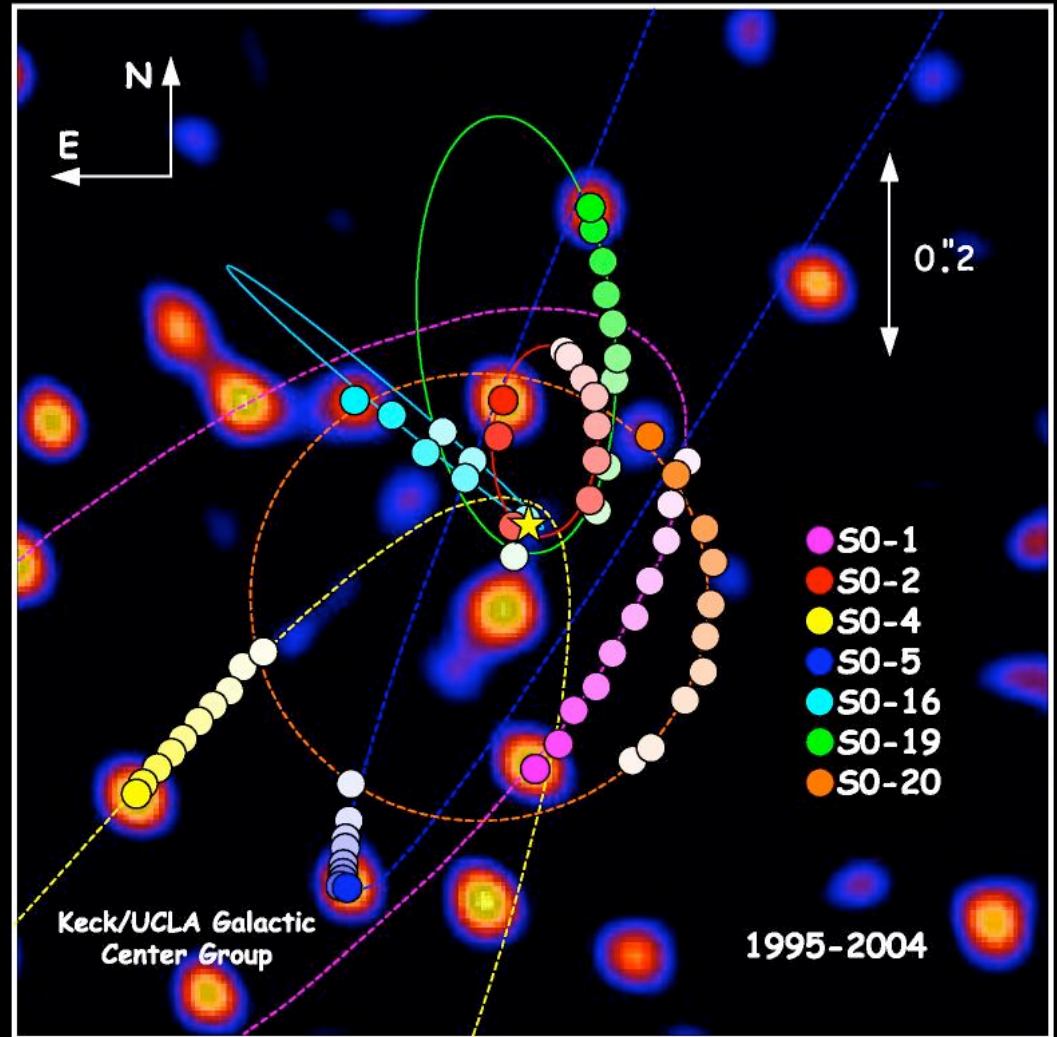
(Haehnelt 1994; Menou, Haiman, & Narayanan 2001; Wyithe & Loeb 2003; Islam, Taylor, & Silk 2004; Sesana et al 2004)

# Center of the Milky Way

Orbits of stars in  
central few light  
days of the center  
of our galaxy.

Apply Kepler's laws  
to these orbits,  
infer mass:

$$M = 3.5 \times 10^6 M_{\text{sun}}$$



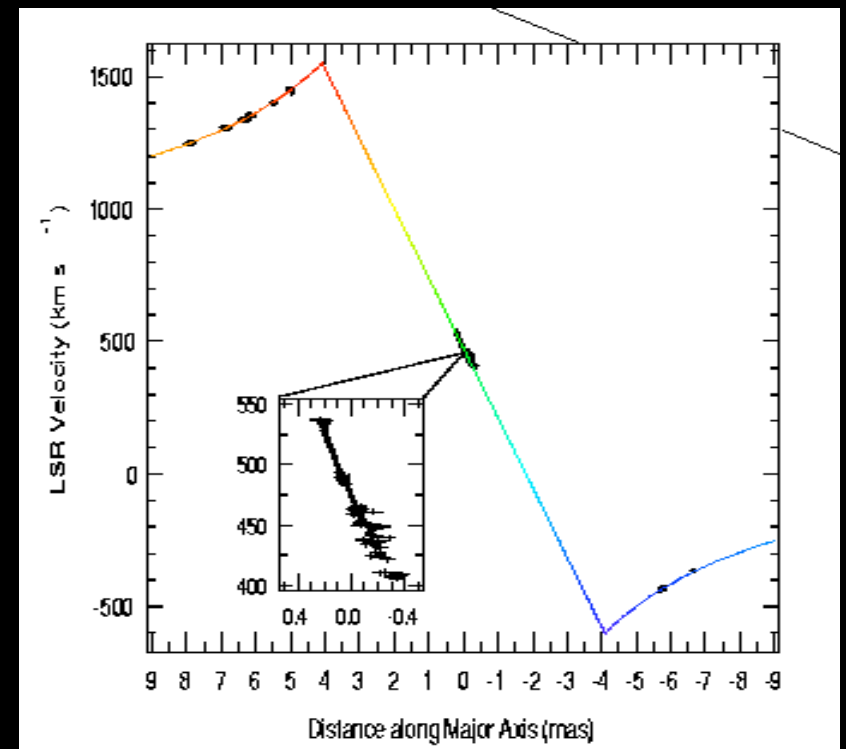


# Center of NGC4258 (M106)

Water maser  
observed in core of  
Seyfert galaxy, can  
use to observe  
orbiting gas.

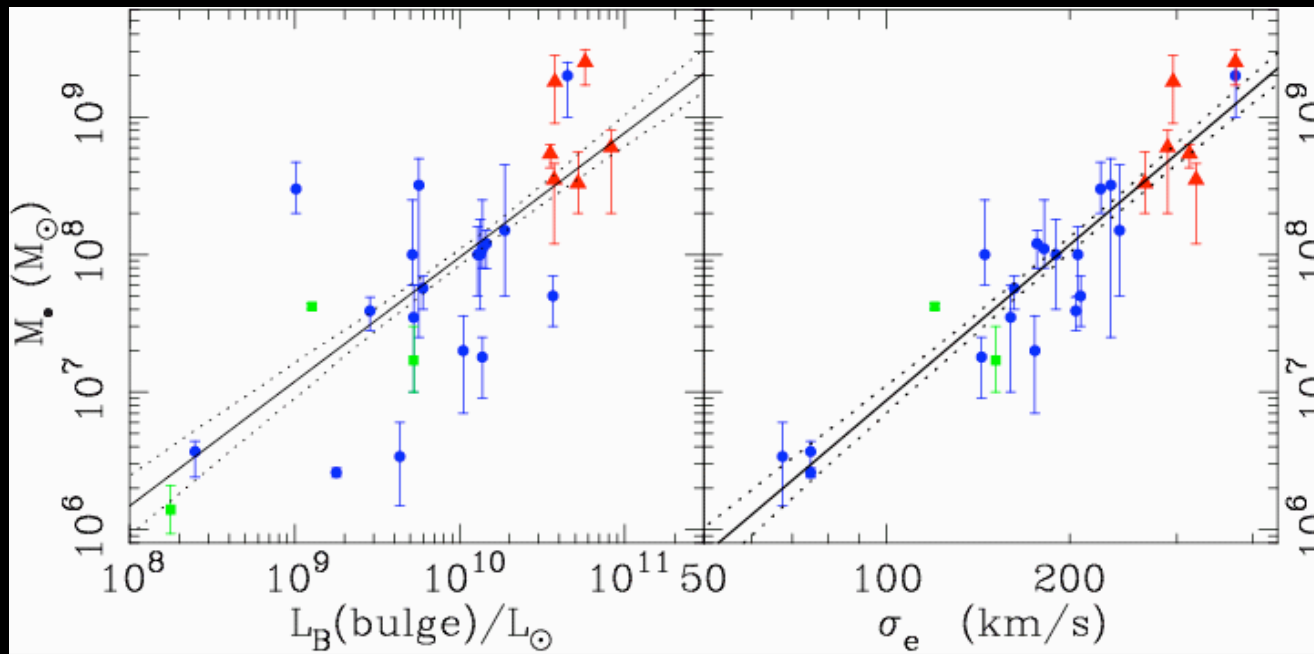
Kepler's law:

$$M = 3.9 \times 10^7 M_{\text{sun}}$$



# Property of black holes strongly correlated to properties of galaxies

Trend: “big bulge” = “big black hole”.  
More precisely: “deep potential well” =  
“big black hole”



$\sigma$ : stellar  
velocity  
dispersion  
in galactic  
bulge.

**Implication: The growth of black holes and galaxies is closely related!**

**Observations and theory: driving us to the conclusion that galaxies (particularly bulges) grow hierarchically ...**

**Natural mechanism to produce binary black holes!**

**Likely that massive binary black hole formation is a relatively common phenomenon, especially at high  $z$ .**

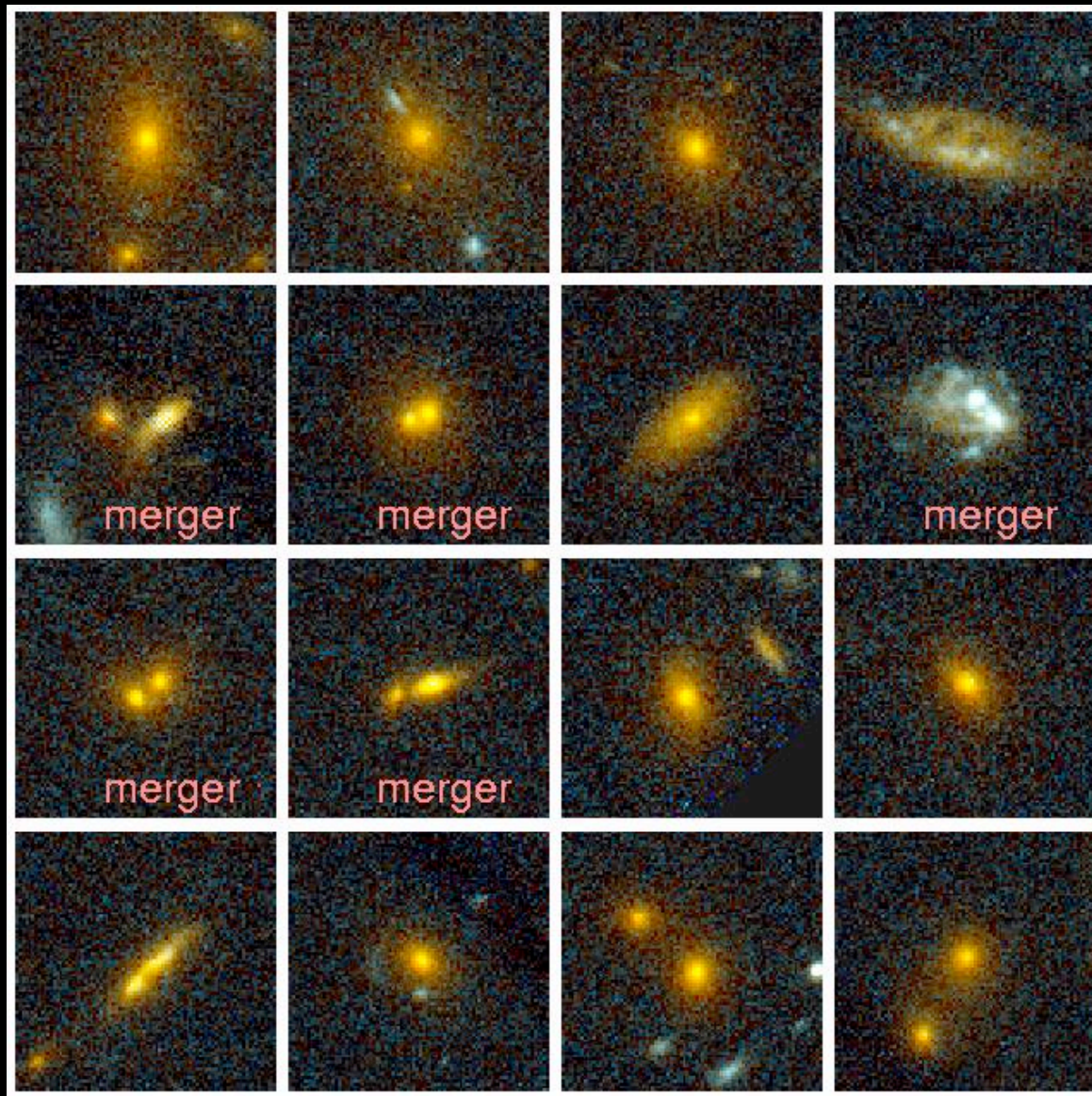
# Action shot: Mergers at high $z$

Mergers in rich  
cluster MS 1054-03  
( $z = 0.83$ )

Shown: 16  
brightest galaxies.

***About 20% are  
merging!***

van Dokkum et al 1999, ApJ  
520, L95.



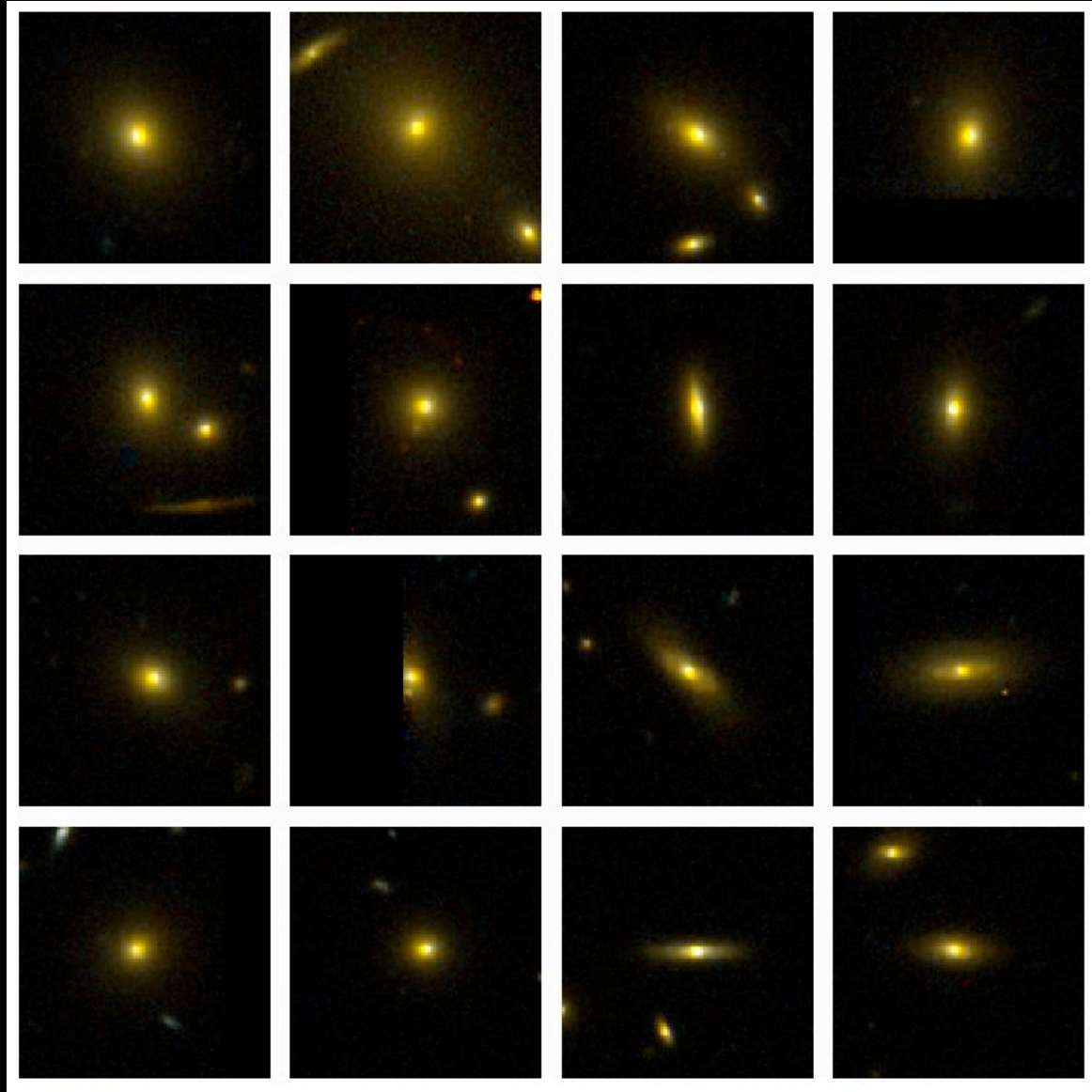


# Inaction shot: No mergers at low $z$

Essentially no  
mergers seen in  
MS 1358-62  
( $z = 0.32$ )

Shown: 16  
brightest galaxies.  
*No apparent  
mergers!*

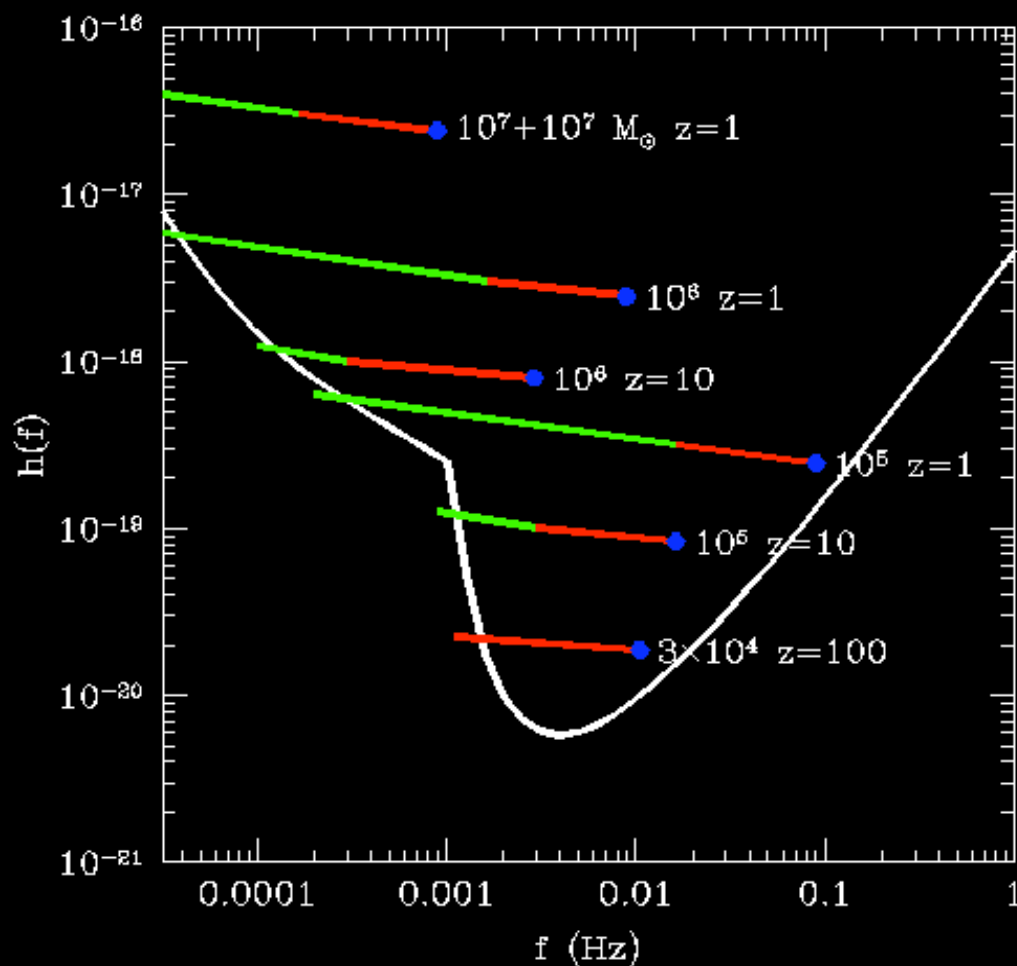
van Dokkum et al 1999, ApJ  
520, L95.



# Binary black hole waves: VERY high SNR!

In relevant mass and redshift range [ $10^5 M_{\text{sun}} < (1+z)M_{\text{total}} < 10^7 M_{\text{sun}}$  out to  $z \sim 10$  or so], integrated signal to noise ratio can be as high several hundred or a few thousand.

**Can precisely measure binary parameters!**

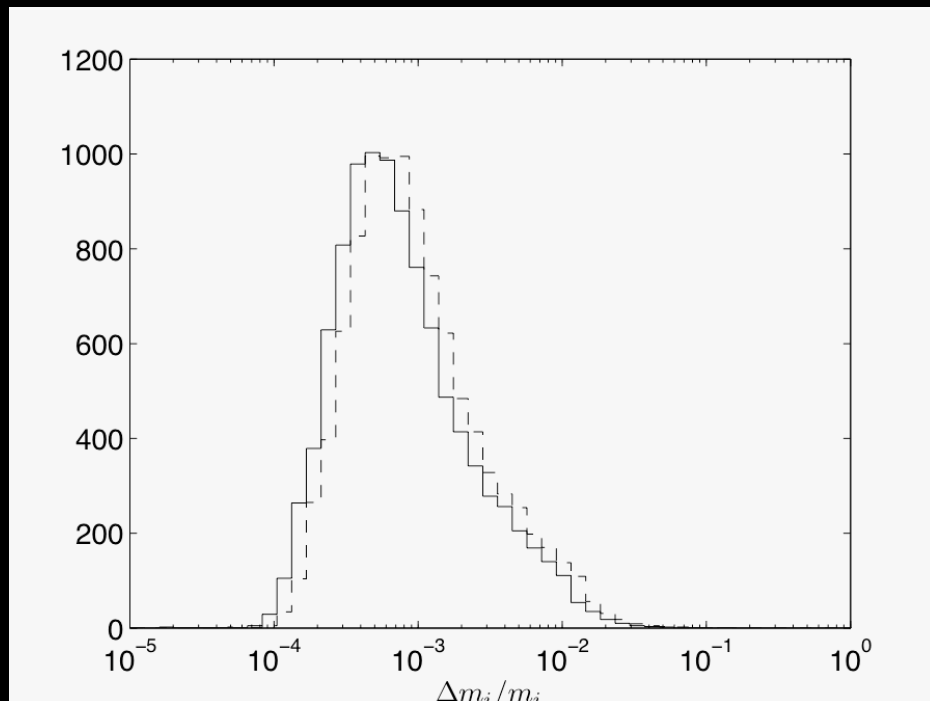


Binary black hole waves  
vs LISA noise spectrum

# What waves measure

**Very accurately determine “intrinsic” parameters which set the orbital phase: Masses of the members of the binary, magnitude of their spins.**

Monte Carlo:  $10^4$  binaries at  $z = 1$ ,  $10^6$  Msun going into  $3 \times 10^6$  Msun; spin precession taken into account, with random spins, spin orientations, and sky positions; errors estimated using maximum likelihood computation of Fisher matrix. (Lang & Hughes, in prep.)

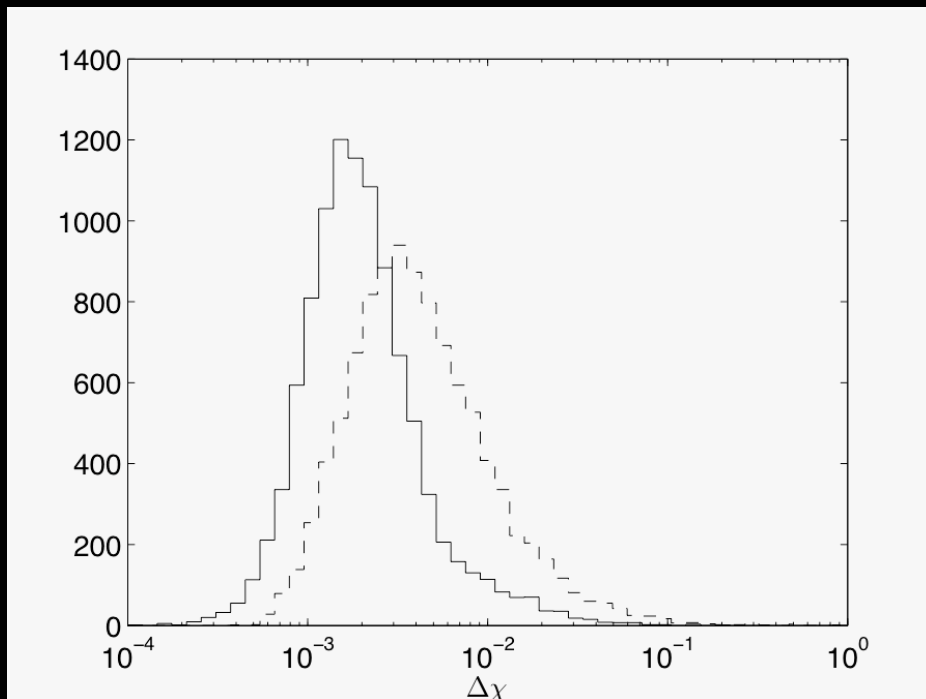


**Individual black hole masses determined with  $\sim 0.1\%$  accuracy.**

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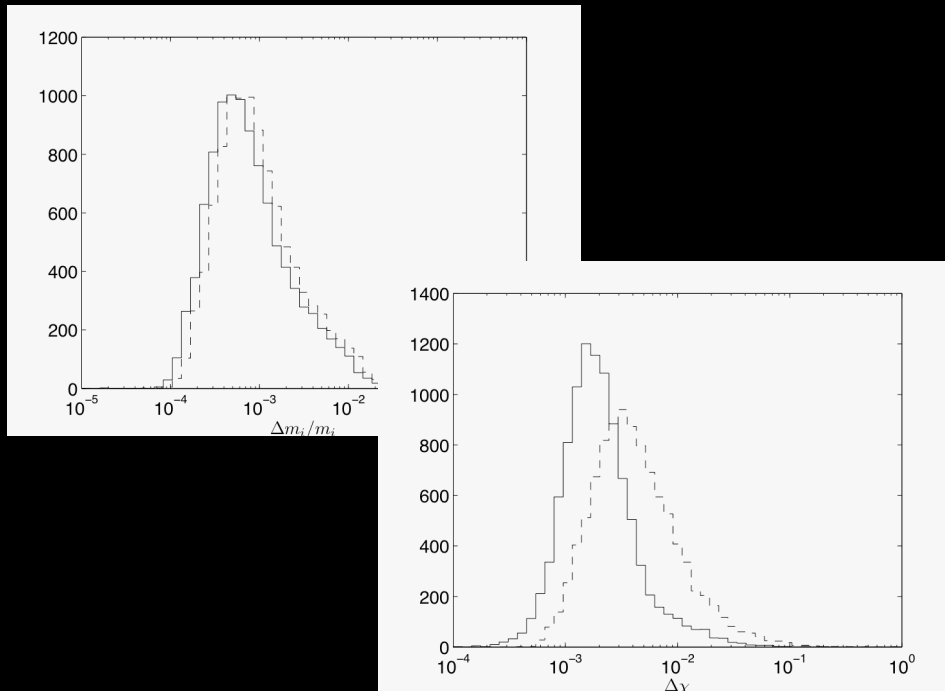
**Individual black hole spins determined with (0.1-1)% accuracy.**



# What waves measure

**Very accurately determine “intrinsic” parameters which set the orbital phase: Masses of the members of the binary, magnitude of their spins.**

Monte Carlo:  $10^4$  binaries at  $z = 1$ ,  $10^6$  Msun going into  $3 \times 10^6$  Msun; spin precession taken into account, with random spins, spin orientations, and sky positions; errors estimated using maximum likelihood computation of Fisher matrix. (Lang & Hughes, in prep.)

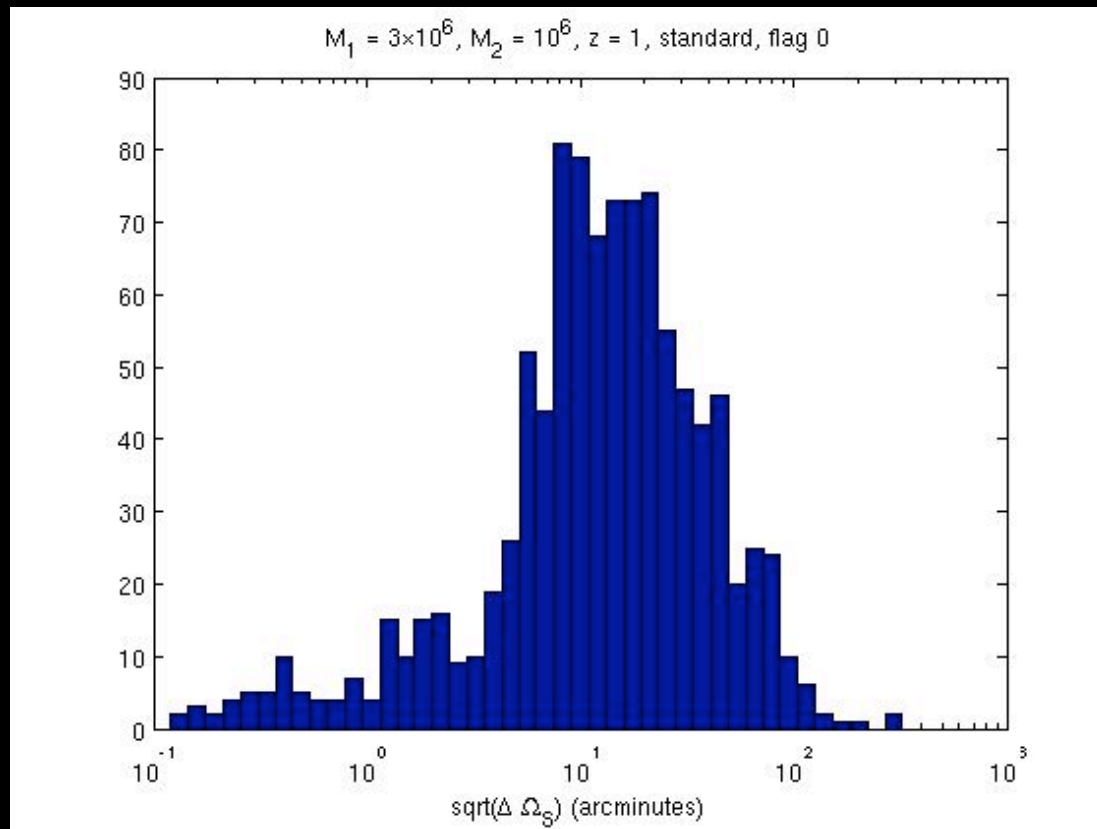


**LISA is a tool for precisely mapping the cosmic growth of black hole mass and spin!**

# What waves measure

Also determine “extrinsic” parameters: position of the binary on the sky, its orientation, distance from the solar system.

Same Monte Carlo setup as for intrinsic parameters.

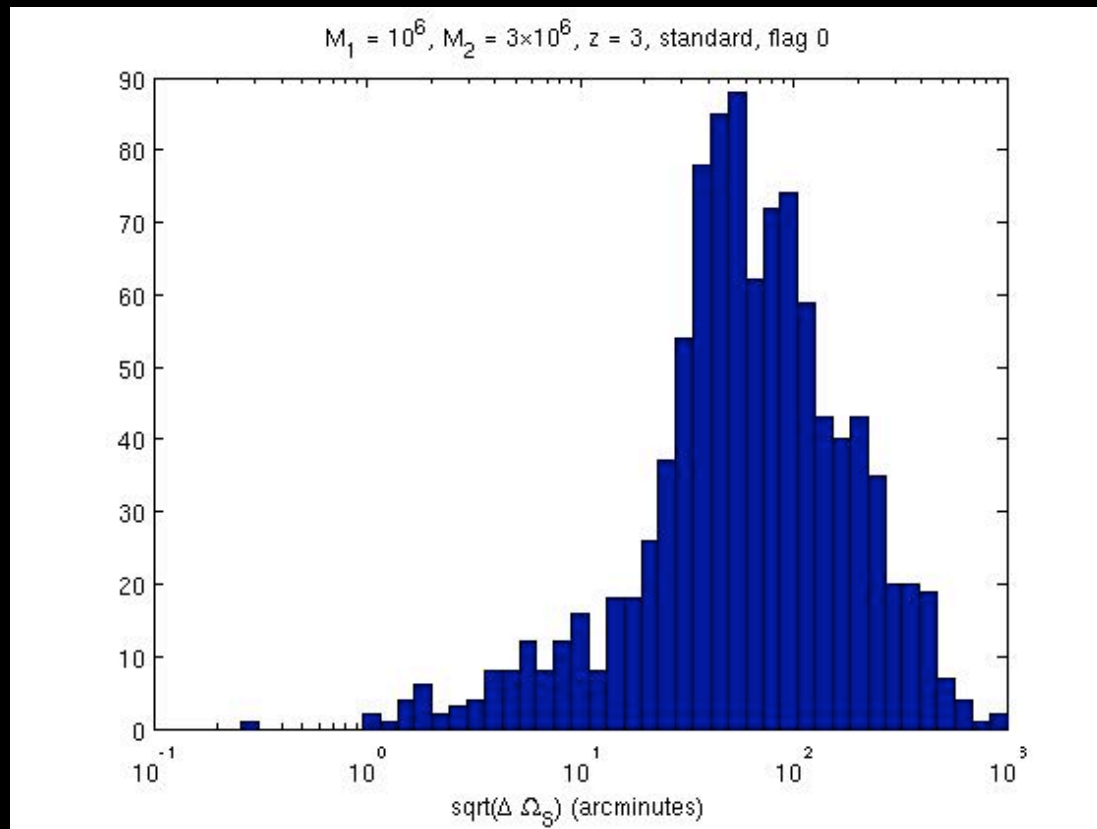


Size of the spot to which binary is localized isn't terrible at low  $z$  - a few to a few 10s of arcminutes.

# What waves measure

Also determine “extrinsic” parameters: position of the binary on the sky, its orientation, distance from the solar system.

Same Monte Carlo setup, but now at  $z = 3$ .

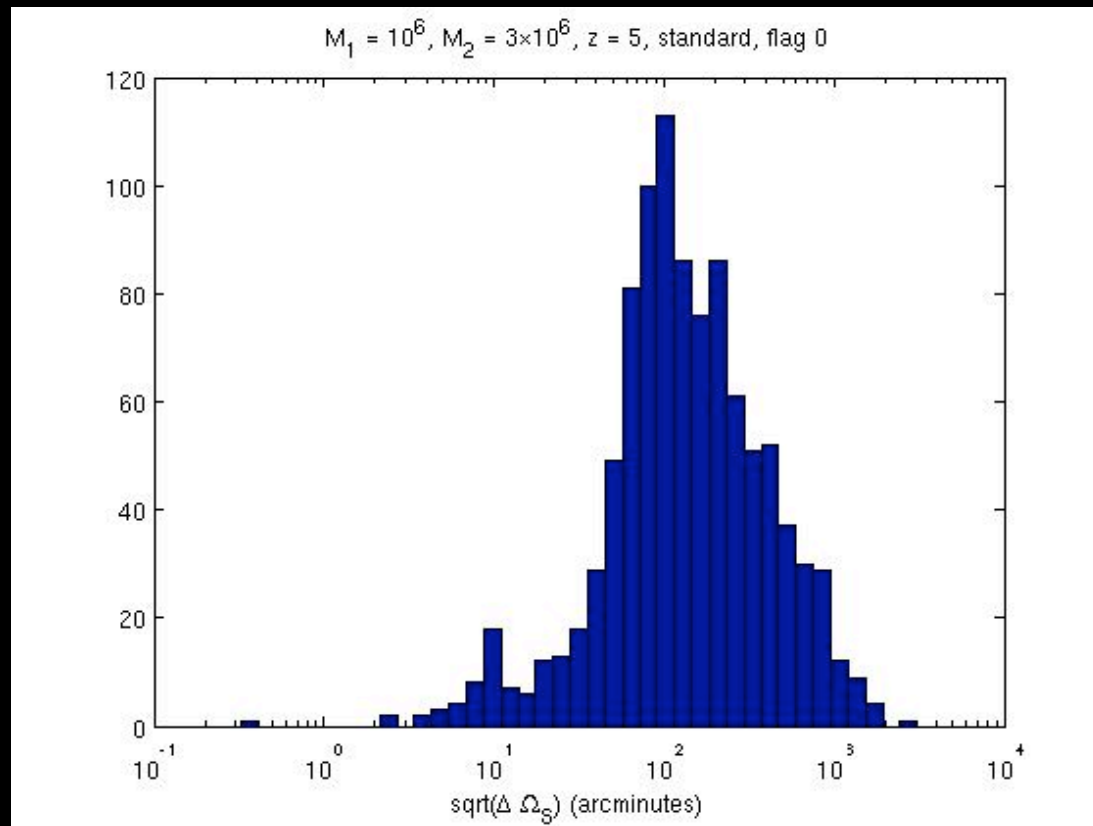


Spot grows considerably as we move to higher redshift: at  $z = 3$ , typically ~3 times larger than at  $z = 1$ .

# What waves measure

Also determine “extrinsic” parameters: position of the binary on the sky, its orientation, distance from the solar system.

Same Monte Carlo setup, but now at  $z = 5$ .



Spot grows considerably as we move to higher redshift: at  $z = 5$ , typically  $\sim 3$  times larger than at  $z = 3$ .

# The LISA “pixel”

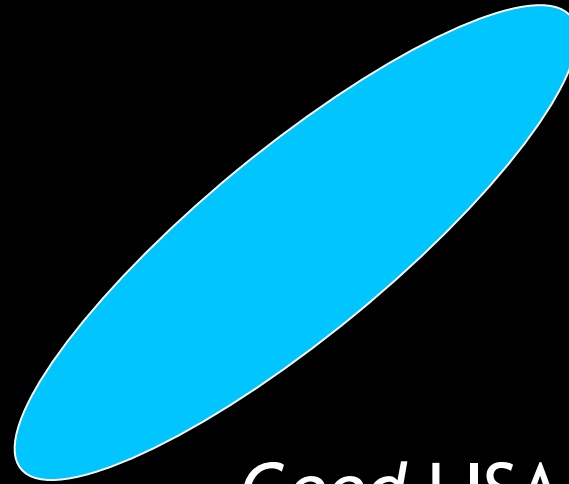


Full moon:  
~30 arcminutes.

# The LISA “pixel”



Full moon:  
~30 arcminutes.



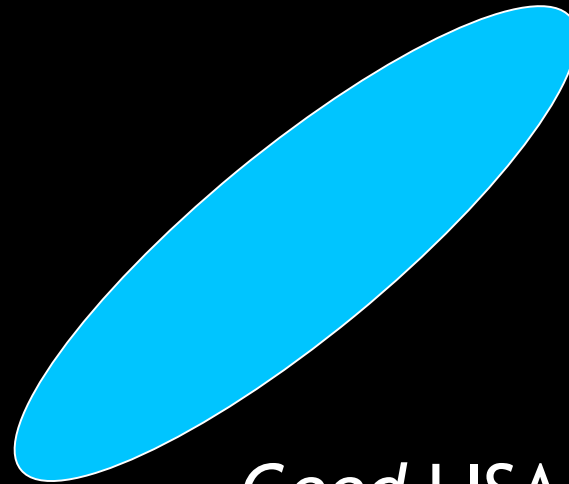
*Good* LISA pixel: A few  
10s of arcminutes  
along major axis,  
~10 along minor axis.



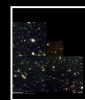
# The LISA “pixel”



Full moon:  
~30 arcminutes.



*Good* LISA pixel: A few  
10s of arcminutes  
along major axis,  
~10 along minor axis.



The Hubble Deep Field:  
144 arcseconds.



Imagine trying  
to find *the*  
galaxy that  
hosts a merger  
in a “pixel” 50  
times larger  
than this!



**Hubble Deep Field**  
ST ScI OPO January 15, 1996 R. Williams and the HDF Team (ST ScI) and NASA

HST WFPC2



Imagine trying  
to find *the*  
galaxy that  
hosts a merger  
in a “pixel” 50  
times larger  
than this!

(Not quite so grim  
since we’ll have  
redshift info ... but  
still will want to  
narrow the search  
space considerably.)



**Hubble Deep Field**  
ST ScI OPO January 15, 1996 R. Williams and the HDF Team (ST ScI) and NASA

HST WFPC2

# Does something go “bang”?

Ideal situation: some kind of electromagnetic signature associated with merger - provides a counterpart, making it possible to pin down location of host galaxy very precisely.

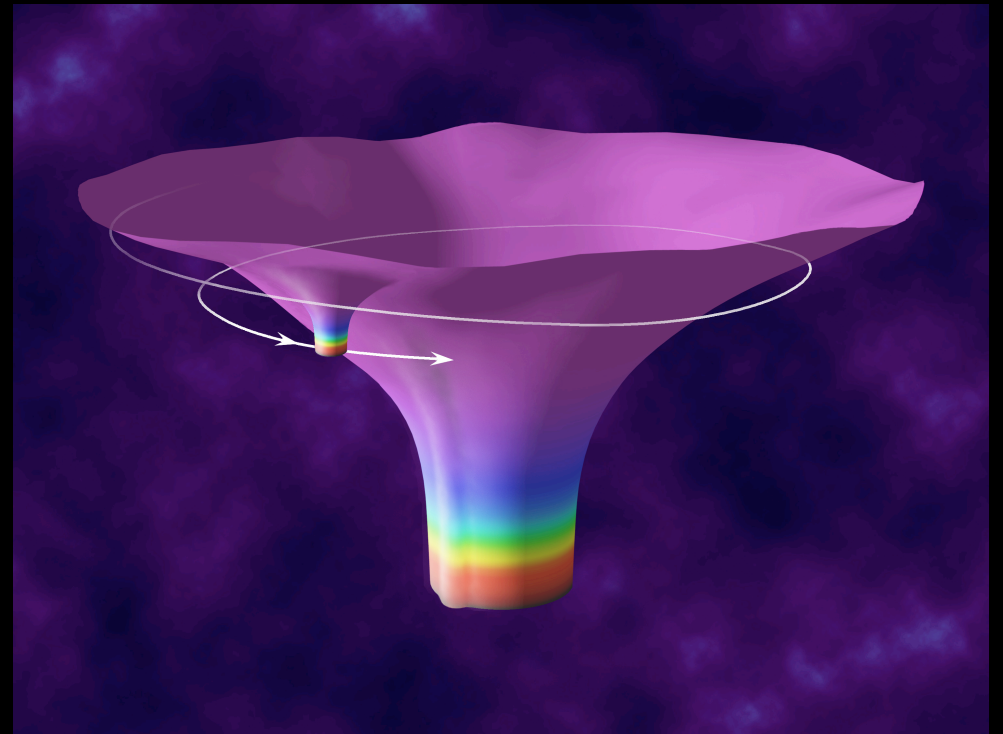
Big question: If some activity happens during the merger process, *does it temporally coincide with the LISA signal?*

If not, might need to just look for morphologically consistent galaxies in the LISA pixel.



# *Really complicated* chirping sources: EMRIs - Extreme mass ratio inspiral

The relativist's favorite source! Extreme mass ratio means spacetime mostly determined by large body: "Clean" probe into black hole spacetimes



Binaries which produce EMRI events formed by dynamical scattering processes in the nuclei of galaxies; talk by Clovis Hopman on Weds.

# The setting (courtesy Marc Freitag)

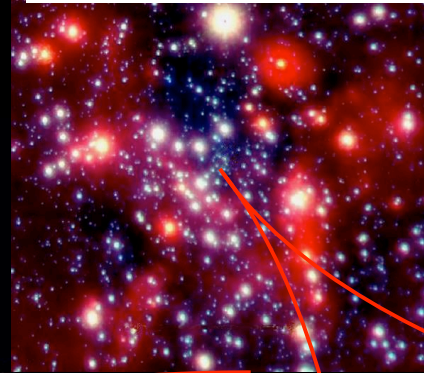
## Stellar cluster



Size	$\sim 1 - 10 \text{ pc}$
Density	$\sim 10^{3-6} \text{ M}_{\odot} \text{ pc}^{-3}$
Velocity dispersion	$\sim 10 - 20 \text{ km s}^{-1}$
Relaxation time	$\sim 10^{7-9} \text{ years}$

## Galactic nucleus

Size	$\sim 1 - 10 \text{ pc}$
Density	$\sim 10^7 \text{ M}_{\odot} \text{ pc}^{-3}$
Velocity dispersion	$\sim 100 - 1000 \text{ km s}^{-1}$
Relaxation time	$\sim 10^{8-9} \text{ years}$



$\times 1000$

$\times 1000$

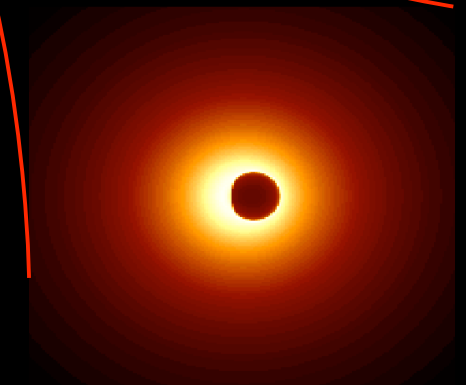
$\times 10^7$

## Galaxy

Size	$\sim 10^4 \text{ pc}$
Density	$\sim 0.05 \text{ M}_{\odot} \text{ pc}^{-3}$
Velocity dispersion	$\sim 40 \text{ km s}^{-1}$
Relaxation time	$\sim 10^{15} \text{ years}$

## Massive Black Hole

Mass	$10^6 - 10^9 \text{ M}_{\odot}$
Size	$R_S = 2GM_{\text{BH}}/c^2 = 10^{-7} - 10^{-4} \text{ pc}$
Rotation	??



# Key issue: Getting compact objects into region of “loss cone”

Region of phase space in which smaller object becomes bound strongly enough that radiation emission dominates its orbit evolution.

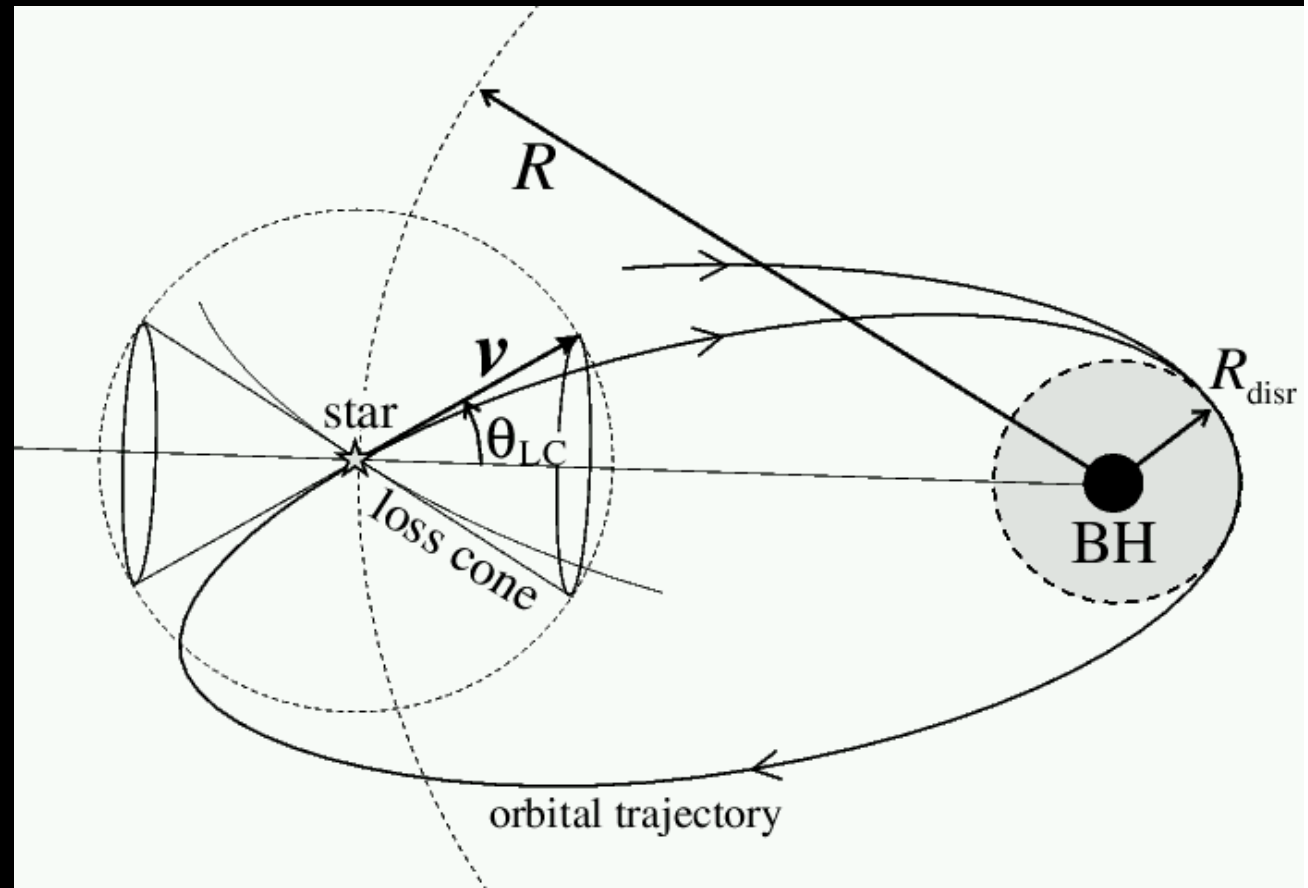
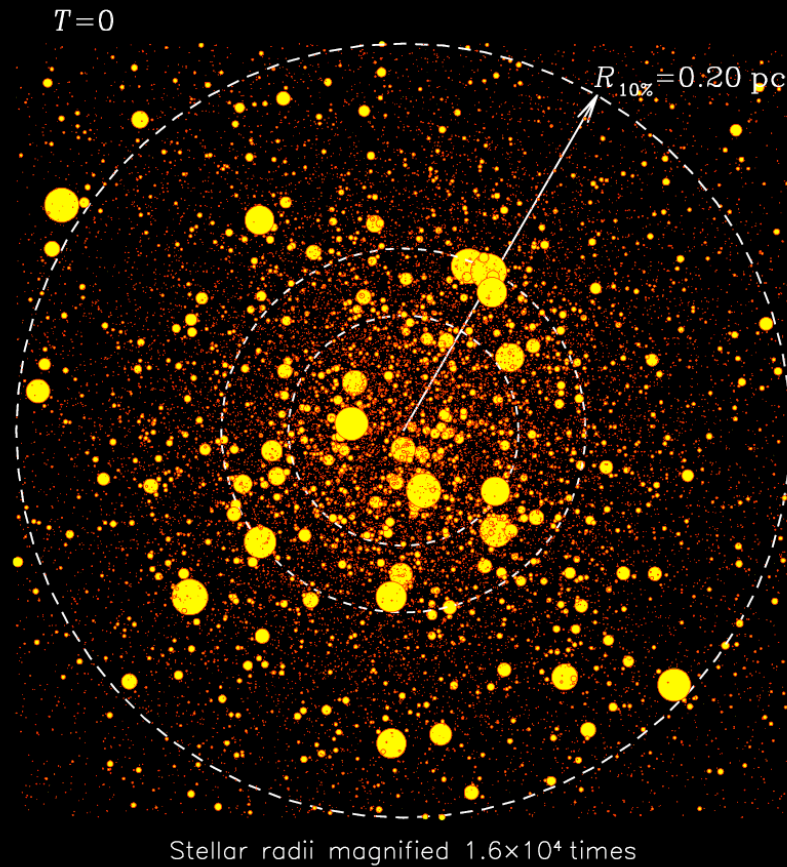


Image courtesy Marc Freitag

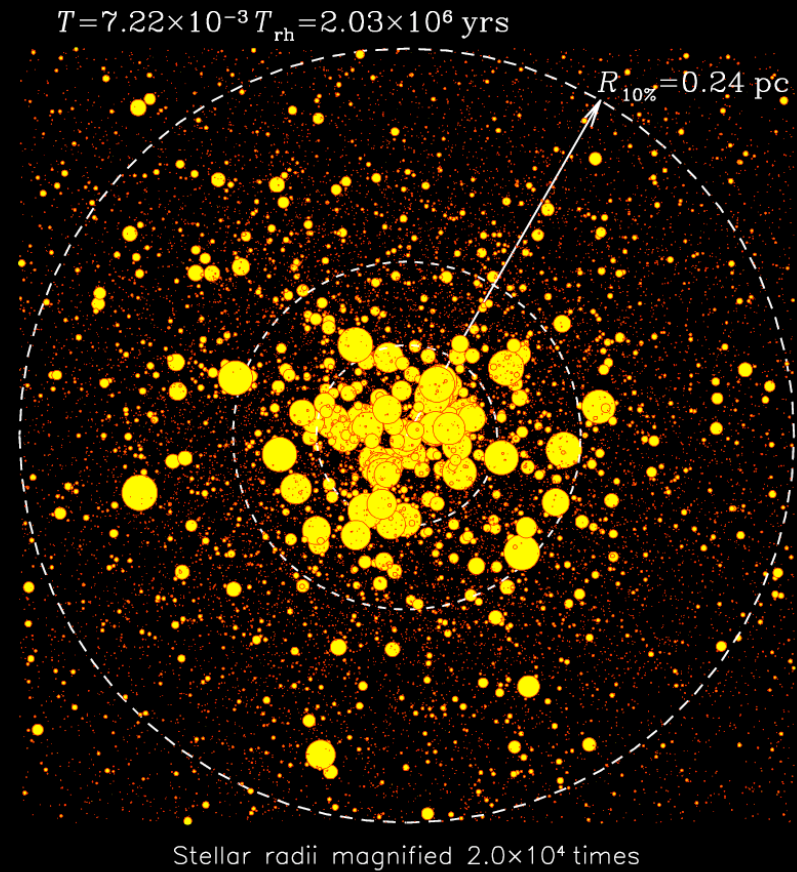


# Key phenomenon: Mass segregation

## Initial conditions



## Core collapse

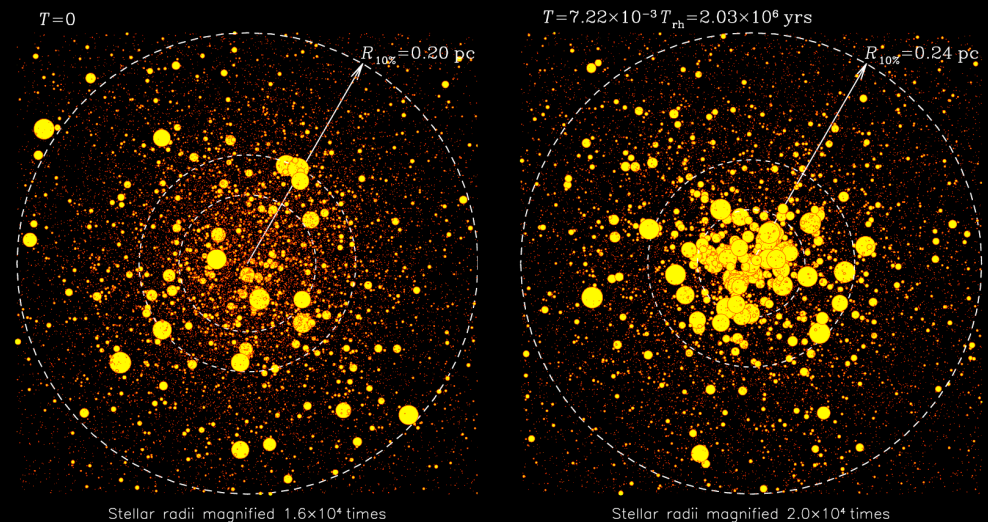


Basic idea quite simple:  
equipartition of kinetic energy;  
most massive objects sink.

Gürkan, Freitag & Rasio 2004;  
Freitag, Rasio & Baumgardt 2005

# Key phenomenon: Mass segregation

For EMRI problem, mass segregation implies that the secondaries which are most likely to be scattered into loss cone are stellar mass black holes.



Means that EMRI events are predominantly stellar black hole captures!

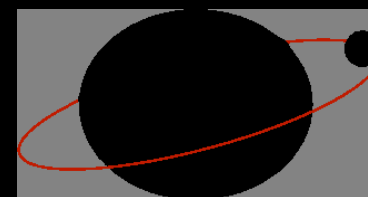
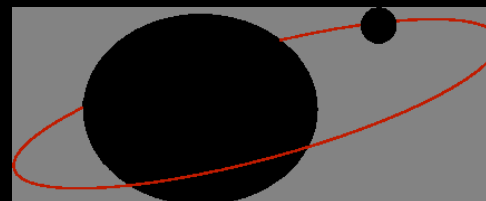
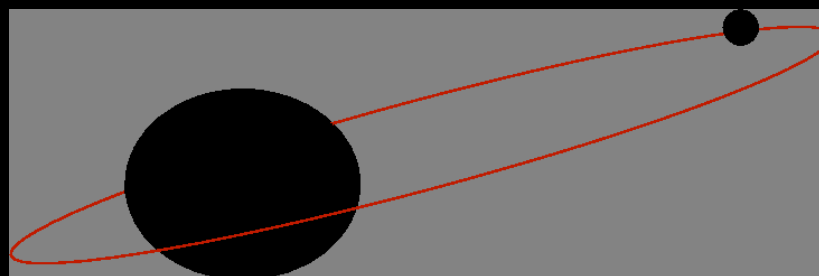
EMRI events should be measureable to  $z \sim 1$ :  
Potentially *very* high event rate.

# *Really complicated* chirping sources: EMRIs - Extreme mass ratio inspiral

Relativity view:

After scattering onto a  
strong field orbit of the  
nuclear black hole,  
smaller body spends ~1  
year spiralling in due to  
GW losses.

Executes  $\sim 10^5$  orbits  
as it spirals in.

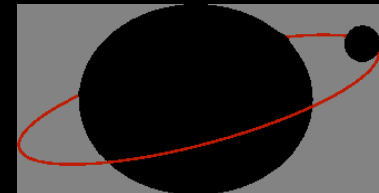
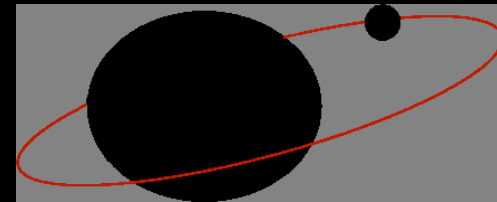
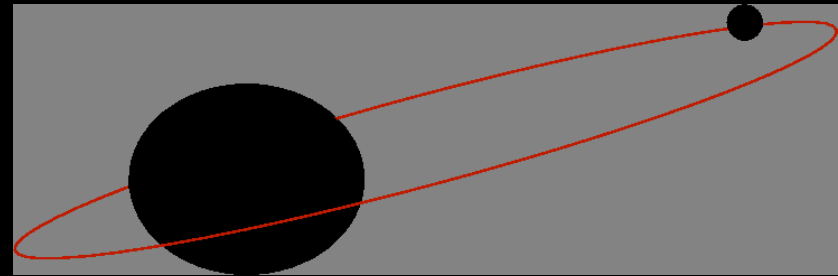




# *Really complicated* chirping sources: EMRIs - Extreme mass ratio inspiral

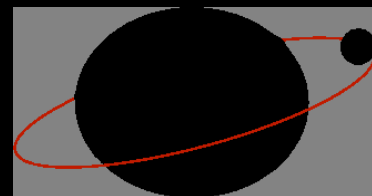
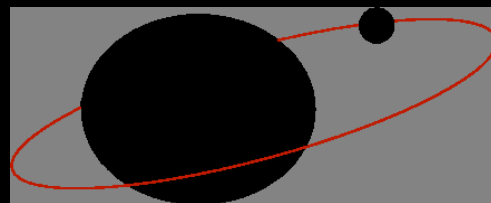
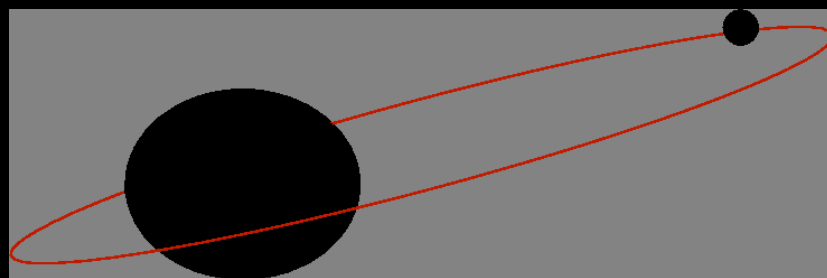
Track phase over those  $10^5$  orbits, can determine character of spacetime with high precision.

Prosaic application:  
Determine mass and spins of quiescent black holes with high precision.



# *Really complicated* chirping sources: EMRIs - Extreme mass ratio inspiral

Track phase over  
those  $10^5$  orbits, can  
determine character  
of spacetime with  
high precision.



$$\delta M_{\text{BH}}/M_{\text{BH}} \lesssim 10^{-4}$$

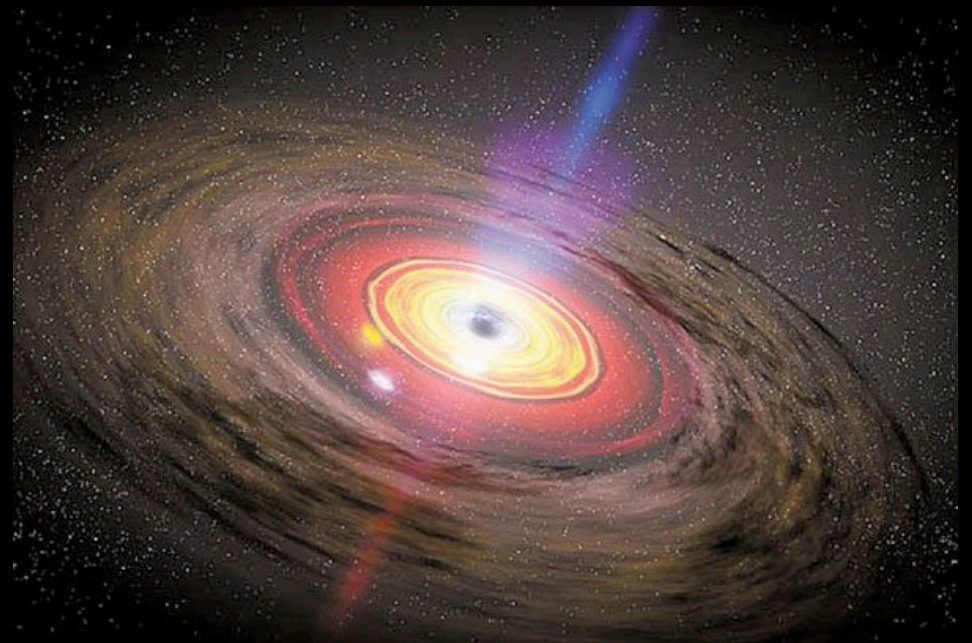
$$\delta a \lesssim 10^{-4}$$

Barack and Cutler, PRD  
69, 082005 (2004).

# *Really complicated* chirping sources: EMRIs - Extreme mass ratio inspiral

More fundamentally,  
test the character of  
the spacetime:  
Weigh its multipoles  
and make sure that  
they obey the  
constraints of the  
Kerr metric.

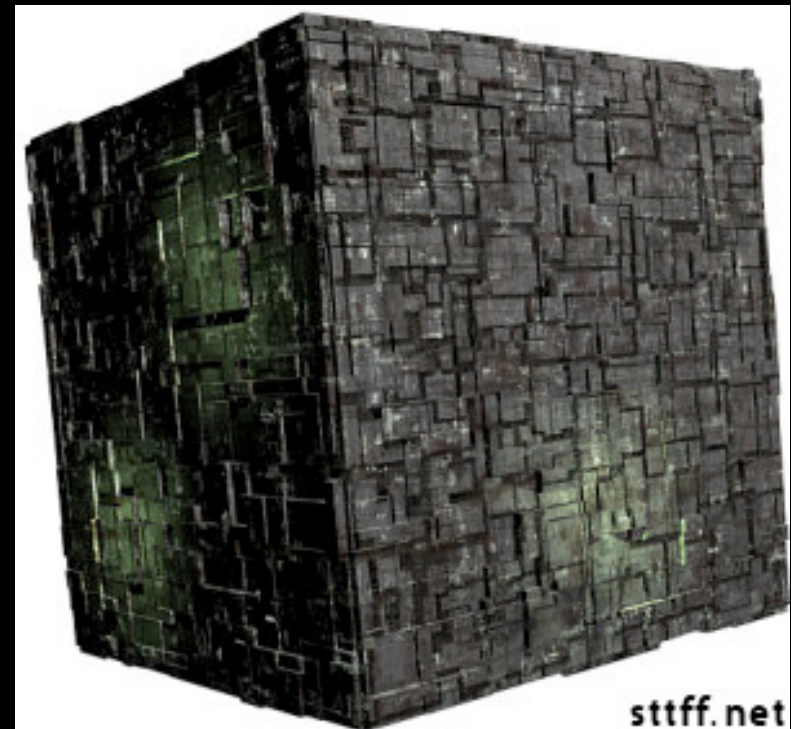
Tell the difference  
between this object:



# *Really complicated* chirping sources: EMRIs - Extreme mass ratio inspiral

More fundamentally,  
test the character of  
the spacetime:  
Weigh its multipoles  
and make sure that  
they obey the  
constraints of the  
Kerr metric.

... and this one:



*Summary:* LISA source astrophysics

Weighing the dark and dense universe!

*My biased opinion*

Key input to astrophysics: Probing the growth of black holes. Circumstantial evidence of mergers is quite strong - a direct probe will open a window onto an aspect of structure growth that cannot be directly measured in any other way.